Beginner's mind and the middle years mathematics student

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Abstract

Being motivated and on task are not enough in themselves for students to engage in meaningful mathematical learning. The Zen concept of beginner's mind describes how one's level of awareness can open one's mind to growth and possibilities. In this case study, two small groups of middle years students who engage in the same mathematical task, one group demonstrating the characteristics of beginner's mind and the other demonstrating expert's mind. While the group with expert's mind focuses on learning how to follow the steps of one proposed solution, the group with beginner's mind explores the task more fully, noticing mathematically salient details about the task that the other group overlooks completely and that help lead to a successful solution. My findings suggest that groups of students who demonstrate beginner's mind are more likely to be absorbed in mathematical tasks, and open to sharing ideas and engaging in meaningful mathematical discussion.

Keywords: beginner's mind, mathematical learning, middle years, awareness, problem solving

Beginner's mind and the middle years mathematics student

Two small groups of motivated, capable students are working on the same math task in the same mathematics classroom. One group reaches a viable solution, the other doesn't. What makes the difference? In this paper, I will explore a form of attention described by the Japanese term *shoshin*, or *beginner's mind*, as it might be demonstrated by middle years mathematics learners. I will use excerpts from a larger study to consider how two small groups of students in a Grade 8 mathematics classroom approach the same problem task differently, with one group demonstrating the characteristics of *beginner's mind* and the other demonstrating *expert's mind*, and to discuss how this affects their performance of the task.

1. Theoretical framework:

Mathematics suffocates from the idea of expertise. In coming across as a big wall of structured ideas, in a way that few other school subjects do, the popular conception of mathematics is that most people will never know enough. Lakoff and Nuñez refer to this as the "standard folk theory of what mathematics is for our culture" (2000, p. 340) or the "Romance of Mathematics," and list a number of its negative effects:

> It intimidates people, alienates them from math, maintains an elite and justifies it. It rewards incomprehensibility, and this inaccessibility perpetuates the romance. The alienation and inaccessibility contributes to the division in our society of people who can function in an increasingly technical economy and those that can't – social and economic stratification of society. (2000, p. 341)

While one could argue in any subject area that you can never know everything, that there's always something to learn, in school mathematics we take this warning to heart, looking to an elite rather than ourselves to guide the way. Consider the women whom Buerk (1982) describes, who have become so caught up in this dualistic view of what mathematics should be that they have lost any appreciation of their own capabilities, and given up on mathematics. As educators, this kind of expectation may also shape how we perceive our own students' abilities. How many times have you heard a colleague wail, "But they should know this by now," bemoaning this lack of expertise.

But what if we reframed this perceived deficit as an asset instead? What if we focused on our students' potential to come to mathematics with what is known in Zen practice as beginner's mind?

> The practice of Zen mind is beginner's mind. The innocence of the first inquiry – what am I? – is needed throughout Zen practice. The mind of the beginner is empty, free of the habits of the expert, ready to accept, to doubt, and open to all the possibilities. It is the kind of mind which can see things as they are, which step by step and in a flash can realize the original nature of everything" (Suzuki, 1985, p. 13-14).

Although beginner's mind shares some characteristics with *growth mindset* (Boaler, 2016; Dweck, 2006) such as the emphasis on learning as a process and errors as opportunities for learning, the focus of growth mindset is on resiliency – why do some students give up quickly while others embrace challenges? Beginner's mind, on the other hand, is about the level of attention, or awareness, of the learner. Beginner's mind is fresh and accepting. Zen practitioners strive for the beginner's mind, recognizing that it keeps them open to growth and possibility, enabling them to give their current activity the full attention it deserves.

It is worth considering what attention is and how it relates to awareness. In every moment we have an infinite amount of stimuli potentially impinging on our senses, too much to take in. Gattegno suggests that people cope with this through what he calls stressing and ignoring (p. 82, 1987), a process which allows us to focus on some stimuli while letting others fade into the background. This focusing is

the act of attention, "the active constitution of a new object which makes explicit and articulate what was until then presented as no more than an indeterminate horizon (Merleau-Ponty, 1945, p.30). It is what makes meaning out of what is otherwise a pile of noise.

> I see attention not just as what puts me in touch with the world of my experience, but what creates and maintains that world. The totality of what I experience *is* my attention. This is meant to include things of which I am subliminally or covertly aware, sometimes through body awareness, sometimes through social awareness, sometimes through emotional resonance, and sometimes through cognitive awareness. None of these need be conscious. (Mason 2004, p. 4)

It is through awareness that we notice the "deviations from expectation create disturbance which can lead to new sense-making" (Mason 2004 p. 3) and learning.

Beginner's mind involves a combination of having a positive attitude about the importance of enlarging one's awareness, a willingness to be open – what Berger describes as making "a choice to look" (1973) – and to be present in the moment. Suzuki writes, "Our 'original mind' includes everything within itself. It is always rich and sufficient within itself. You should not lose your self-sufficient state of mind. This does not mean a closed mind, but actually an empty mind and a ready mind. If your mind is empty, it is always ready for anything" (1985, p. 21). This idea of the mind being both rich and sufficient but also empty may at first seem to be contradictory. What the mind is empty of is expectations and judgments about how things should be, thus the mind is ready to take in what's around it, to explore, to notice. It is aware, and through this awareness is the potential for movement and growth. As Gattegno once wrote, "Only awareness is educable" (1987, p. 220).

As Lakoff and Nuñez have noted, in popular culture mathematics has become closely linked with the idea of expertise. Perhaps part of the reason for this is how "amazingly compressible" (Thurston, 1990) mathematics is. Consider how much is packed into the acronym "SOH CAH TOA", a reminder of how the ratios inherent in triangles are related to angles that a student might scribble on the margin of a trigonometry test page. As Tall writes, "The shift from process to concept is an enormous compression of knowledge allowing us to replace thinking about a process in time to think about a concept as a single mental entity that can be manipulated in its own right" (2013, p. 46). The benefit of this is that it "leads to automatisms which make awareness available for other realms and for the use of one's past at a minimal cost in energy. To tell what I remember costs me no expense of what I need to be aware" (Gattegno, 1987, p. 90)

What is a benefit in one sense, however, may be a drawback in another. When there is a push to "cover the curriculum" in a certain amount of time, concepts may be presented by the teacher, or received by the student, or both, as a compressed *fait accompli* rather than as an area of inquiry and exploration. Thus expert awareness is translated into routine and mechanical behaviour (Mason, 1994).The lesson turns into "this is the method we must use", or "just follow this acronym" and thus we get students knowing they need to use "FOIL" (First, Outside, Inside, Last) to multiply binomials without necessarily understanding the mathematics compressed in this magic mnemonic. What is meant to save time and energy ultimately only serves to hinder awareness. In expert's mind, efficiency trumps understanding.

The Romance of Mathematics can be tempting for students because of its certainty: answers are either right or wrong, there is a specific set of steps to follow, and the concepts must tackled the particular order set out in the textbook. For many, mathematics means following a script, a preordained path, and we can see this in the questions that students often ask: "Am I doing this right?" "Have I forgotten a step?" Students can congratulate themselves on mastering the "correct" steps, without any concern about what else could have been, and in this way the expert's mind becomes constricted with ideas of how things "should be." Composer Debussy once wrote, experts may have "neither the will nor the courage to break with their successes, failing to seek new paths and give birth to new ideas" (Romesburg, 2001, p. 239-240). This can result in the repetition of old ideas rather than the embrace of the flux and possibilities of the situation before them. As Suzuki puts it, "In the beginner's mind there are many possibilities; in the expert's mind there are few" (1985, p. 21).

In this article, I will discuss two groups of motivated, on-task students working on the same task in the same classroom in the spring of the school year. Although, both groups initially pursue a similar solution path for the task, one group's beginner's mind allows them to be aware of and open to ideas that cause them to veer off that path to develop a new one and that makes all the difference.

2. Methodology

The excerpts I will discuss in naturalistic case study were drawn from a larger study of collective behaviour in mathematics classes that took place in a suburban school in the Lower Mainland of British Columbia. Two mathematics classes of Grade 8 students, taught by the same experienced teacher (Mrs. Shug), were each split into small groups to work on "Problem of the Day" tasks for 20 - 25 minute long sessions. These took place roughly every week for five weeks in the spring of the school year and made up the "Problem Solving Unit" for both of these classes.16 of the 30 students in each class had volunteered to be in the study, and while all students in the class participated in the problem-solving tasks, only study participants were video and audio-recorded.

The middle years age group is known for its high energy and enthusiasm for socializing, making its members well-suited for group work. The composition of each group was chosen by Mrs. Shug and me according to who we thought would collaborate well and be willing to talk about mathematics together. During each session, Mrs. Shug provided each group with two copies of the task sheet and provided a brief introduction to the task so that students could ask any general questions about what was expected or any unfamiliar vocabulary. During the task, Mrs. Shug only interacted with groups in the study if they had a specific question for her, or by asking them to describe their solution to her when they appeared to be finished. She tried to avoid any discussion about whether a group had a correct solution or not, in order to encourage students to rely on their own ideas and reasoning. Students were expected to "show their work" on the task sheets, although not to create a good copy, and these sheets were handed in at the conclusion of the session. Towards the end of each session, Mrs. Shug led a whole class discussion where groups were randomly selected to describe their solutions to the rest of the class. If no successful solutions were reached, Mrs. Shug would describe a possible successful solution to offer some sense of closure for the students.

This paper focuses on two groups in one of the classes: NIJM (Nitara, Ian, Jacob and Michael) and EGL (Eliana, Geri and Lucy. The fourth member of the group was absent from the Snow Day session). This was the first time these particular students had worked together in groups in the study, and this was the second task in the study. The "Snow Day" task was selected by Mrs. Shug from several possible tasks that I offered her and is originally from the Math Forum website (mathforum.org) although the version used in the class included the names of actual teachers at their school (which I have replaced with pseudonyms for the purpose of this article)[1].

Snow Day

> It hasn't happened in a long time, particularly at this time of year, but if it ever snowed hard enough for long enough, school would close for the day. Yes, you'd be sad but you'd get over the pain in time. Anyway… School closings are announced on local TV and radio stations, but sometimes you have to watch or listen for a long time before they announce your school. Mr. T would get advance notice of any closings. He then calls Mrs. J and Mrs. W. Mrs. J calls Mr. M and Mrs. G. Mrs. W calls Mrs. S and Mr. B. The phone tree continues with Mrs. S calling Ms C and Ms A, and Mr. B calling Mrs. V and Mr. K, and so on.
>
> We are going to assume that all the teachers want to get back to sleep as soon as possible, so they are not going to chat, and each call will take only one minute.

[1] All student and teacher names in this article are pseudonyms.

> One morning Mr. T finds out that there is no school and makes his first call at 6:00 a.m. Ten minutes later, at 6:10 a.m., everyone on the phone chain is back asleep. How many phone calls have been made?
>
> What time will the last calls be made if the phone chain is extended to include 1,000 teachers in the school district?

I follow Powell et al in seeing the "development of mathematical ideas and reasoning as complex and non-linear processes" (Powell, Francisco, & Maher, 2003, p. 413) and have used their method of data analysis, which follows a sequence of seven interacting, and non-linear phases. 1) The first phase was an attentive viewing of the video data and listening to the audio data. As the sessions took place in a noisy middle school mathematics classroom, where the whole class was engaged in doing the task, the video-recorder's microphone was not sufficient to pick up all of a group's discussion and I placed an audio recorder on a desk in each participating group's working area to supplement the video recording. 2) I then wrote a coarse-grained account of what had happened. 3) In this phase I identified critical events: "critical events are similar to what Gattegno (1987), observing learners doing mathematics, calls moments of awareness and that these events or moments often compel researchers to reflect on their antecedent and consequent events" (Powell, Francisco, & Maher, 2003). For this analysis, I used Hewitt's idea that "Awareness is revealed not through actions themselves but through making explicit what is guiding those actions" (2001, p. 41). As group work is necessarily public, with members striving to share their understandings and ideas with each other (Engeström, 1994; Stahl, 2006), observation of the collective discourse of small groups can provide an indication of to what group members are attending. As Gattegno (1990) wrote, "All I have to share is my awareness of my awareness." Thus, I defined critical events as episodes where group members were explicitly discussing the task and their unfolding solution paths – for instance, asking and answering each other's questions, sharing and debating ideas, and discussing the task itself. 4) I transcribed what the participants said during these critical events, also noting major gestures and physical actions (for example, one group member passing a task sheet to another). 5) Coding emerged through an iterative process of considering what participants appeared to be aware of, using a constant comparison method (Glaser & Strauss, 1967). For instance, if one of the groups appeared to attend to a certain aspect of the task, I myself was attentive to whether or not this same locus of attention was revealed elsewhere in the session. I viewed the video and listened to the audio separately, also played the audio and video recordings simultaneously, and reread the transcript. I then went on to observing the other group's audio, video and transcript, to see if the group demonstrated the same locus of attention, and if so what unfolded. 6) I constructed a storyline for each group and 7) composed narratives.

3. Results

In this section, I offer a narrative of each group's session to give a sense of each group's awareness as the students work on the task. I begin with Eliana, Geri and Lucy (EGL), whom I believe demonstrate expert's mind, and then I describe the session of Nitara, Ian, Jacob and Michael (NIJM) whom I believe exhibit beginner's mind.

[FIGURE 1]

3.1 EGL: "So, what do you do then?"

Geri begins working on the Snow Day task sheet shortly after it is handed out, a few minutes before Mrs. Shug, the classroom teacher, even introduces the task to the class. Geri quickly identifies the task situation as being an exponential one (one person phones two people, each of those two people phone two people, and so on and so on) and moves through her calculations within a few minutes. She then pulls out a workbook from her regular math lessons and starts completing on exercises in that.

Sitting across from Geri, Eliana and Lucy work on the task together. After sketching out an initial tree [Figure 1] and reading the task aloud carefully, they appear to be stuck.

Lucy: I'm just guessing but it's one minute isn't it ten calls?

Eliana: I thought so too but then there must be a trick because…

After some brief pondering, they turn to Geri

Eliana: Did you get it? The first question?

Geri flips over her sheet and begins sketching out the method she has been using, talking them through the steps she used. She doesn't give them hints or nudges; she flat out tells them what to do and what the answer is.

Lucy: So you're just timsing by 2? Oh, I get it!

Eliana and Lucy begin working together again.

Eliana [to Lucy]: "So, what do you do then? Like, what did you do?"

Lucy: So the answer to how many people… I don't know

It appears that while Lucy recognizes that the pattern of numbers Geri has been working with involves doubling, she doesn't understand why Geri is using those numbers. The girls turn again to Geri, who explains some more.

Eliana: Oh, oh. She means that each person calls in a minute.

Lucy: Oh

Eliana: But they will call at the same time, right? But then, how do you know if it's like he calls these 2 people in one minute or calls one person in one minute and then one person in one minute?

Geri: Each call is one minute [inaudible]. 3 minutes, 4 minutes. Right? 5 minutes, 6 minutes [she continues counting]

Eliana and Lucy look at their sheet and Eliana starts writing again.

Eliana: I got, I got, okay. 1 2 1 2. Okay. I got it. Remember? Okay.

As Eliana writes, Lucy watches, and Geri is now erasing the front of her own sheet where she did her original calculations. She erases this work completely.

Eliana is counting aloud and reaches 35. She looks at Geri.

Eliana: Is it 35? For the first question?

Geri's response is inaudible but apparently she did not get 35. Eliana counts again and seems to be happy with her new total.

Eliana [to Geri]: So then what do you do then? How do you start?"

[FIGURE 2]

Geri shows Eliana how to start, using the work on her own sheet [Figure 2] as an example. She returns to writing in her math workbook while Eliana and Lucy are talking about the doubling pattern. They borrow Geri's sheet to compare it to their work. Eliana asks Geri a couple of questions about what's on Geri's sheet and then returns the sheet to her. Geri's sheet continues to be passed back and forth for the remainder of the time that they work on the task.

Lucy starts writing in her own math workbook as well, while Eliana continues working on the Snow Day Task. After a couple of minutes, Lucy stops doing the workbook and looks over what Eliana has written, comparing it to Geri's sheet. Geri grabs her sheet back, adds more detail, correcting a calculation, and then hands it back to them. There is a discussion about what the group will say about their solution when Mrs. Shug starts the class discussion. Eliana says, "I don't get it". Geri explains the writing that she recently added to her own sheet, while Eliana and Lucy observe. Geri then pulls Eliana and Lucy's own sheet away from them and starts writing on it (Figure 3 – her writing is at the bottom of the sheet) while she talks and Eliana and Lucy observe. Then she returns their sheet to them. Eliana takes Geri's sheet and compares it to what Geri has just written on her and Lucy's own sheet.

[FIGURE 3]

Geri: This one is not what I added. These two are.
Eliana: Oh…. When you add all this
Geri: Yes
Eliana: Okay. I got it.
Lucy: I don't really get it. I don't have time.

Lucy, who is dressed in her school band uniform, explains how she's nervous about a band performance later that day. Eliana puts their task sheet away and all three girls discuss a book assignment they have due. Geri says she's read the book already and it's boring. Eliana begins to read the book, while Geri and Lucy do other work. A little later, Mrs. Shug notices that they're done and stops by to ask them about their solutions. Eliana very briefly explains what she and Lucy have written on their sheet. Geri then pulls out her sheet and gives a longer explanation about the patterns with which she was working. Mrs. Shug says, "Well okay then," not commenting on whether or not the solutions are correct. She hands them some integer subtraction practice sheets to work on until it's time for the whole class discussion. Both Geri and Eliana smile as they put away their task sheets and begin working on the integers sheet.

The group discussion is like a question-and-answer session, where Eliana and Lucy question Geri about her method. Geri does not actively try to dominate the other two members of her group – although she keeps an eye on what they are doing, she rarely offers advice unless specifically asked for it – but it appears that Geri's influence holds a lot of sway over how the other girls approach this particular task. During the session, Eliana and Lucy frequently use Geri's sheet to check the accuracy of their calculations. At one point, Geri even takes the sheet where Eliana and Lucy's sheet and writes on top of their work. Late in the session, Eliana's questions appear to prompt Geri to write more on the back of her sheet but this is only to make her written work clearer and provide further detail. No new ideas are added, though, and her solution strategy does not change. During the class discussion at the end of the session, it becomes clear that this group has missed fully understanding how the telephone tree works in the Snow Day task and that Geri's strategy is actually incorrect. There is little reaction by the girls to this news. They hand in their two task sheets without further comment and prepare for the next classroom activity.

[FIGURE 4]

3.2 NIJM: "If we counted it, we were right!"

The members of this group start reading the task sheets as soon as Mrs. Shug has handed them out. After Mrs. Shug introduces the task to the class, the group begins working in earnest, initially in pairs (Nitara with Jacob, Ian with Michael) with each pair drawing out the tree on their own sheet (Figure 4 is from Nitara and Jacob's sheet). Michael immediately suggests to the whole group that the answer is not 10 but 14, and they continue working as a whole group from that point on, although writing and drawing continues to occur on both task sheets. As Ian and Nitara each draw, they both note that as the calls continue some of the calls are occurring at the same time as others. Ian says that it's 4 minutes for 6 people to call. Michael asks if this means the ratio is 6 to 4, but this idea of a ratio is dismissed by Ian, who continues drawing out the tree and counting. There is a discussion about whether the calling stops once they've run out of the teachers' names listed in the task, but then they note that the task says "and so on". They wonder how many teachers work at their school, and Jacob gets a list from a page in his school planner to find out. In the meantime, Ian has been reading aloud from the task, ending with the first question, "How many phone calls have been made?"

Nitara: I would say 10 because each one is a minute. No, more than one is happening at a time.
Ian: Watch, watch. The first six people who are called, it only takes 4 minutes because she's calling one and she's calling one and she's calling one and she's calling one again.
Michael: 2 minutes each.
Ian: Mr. T is calling 1 minutes 2 minutes, so 3 minutes 4 minutes because they're calling at the same time. So then from here it'd be 5 minutes, 6 minutes, 7 minutes, 8 minutes. But there's going to

be another chain here so now 4 are being called at the same time and then 8 are going to be called at the same time and then 16 are going to be called at the same time. Continuing on. So it would only take one minute to phone.

Michael: She sees clearly now.

Ian: Probably not. [to Nitara] I mean, do you understand?

Nitara: Kind of, yeah.

Ian decides, "Okay, I'm going to make a better graph" and asks everyone to watch as he draws out a new version of the phone tree. It proves to be tricky, and he has to recount a couple of times. There is group discussion about whether or not the original call to Mr. T should be counted and they decide that it shouldn't. They come up with 34 as a possible answer for the number of calls made.

They then begin working on the second part of the task: how long it would take for 1000 calls? Nitara suggests that they multiply by 1000 but this idea is not taken up by the group. Ian starts yet another tree and again he has difficulty keeping track of the count with the tree and has to keep restarting. This time he ends up with 32, then extends it to 64. Nitara suggests that maybe they could try working backwards from 1000, but there is discussion about how they don't know how many minutes they'd be working with and they don't have a calculator. Ian continues drawing the tree and trying to keep track of the minutes. He's frustrated and doesn't appreciate Michael's attempts to lighten the mood with his wisecracks. Jacob actively monitors the tree pattern and Ian's arithmetic. Nitara then takes over the adding of minutes from Ian who takes a break. Jacob wonders if 1000 calls would include all of the teachers in their own school district. There is some discussion of that and about if the answers that are coming up make sense for that situation.

At this point Mrs. Shug comes over to see how the group is doing. Ian explains the doubling pattern, and how they're extending their "graph" and "just adding the pattern up" but he's concerned that they've now gotten to 1120 people which is more than 1000 and that maybe this current tree is missing some people. Mrs. Shug asks how they solved the first question of the task and both Nitara and Ian explain how they used "a graph" to solve it. Mrs. Shug nods and leaves. As she walks away, Ian calls out "Were we right?" Mrs. Shug replies, "You've got one of the numbers I've heard [from other groups in the class]."

The group discusses who is going to present their task solution to the rest of the class. Nitara wants Ian to present because "I didn't understand half the heck of what you were doing." Ian suggests, "We could each see each other's methods I guess. Whoever has the best one…" At this point, Nitara grabs the sheet he's been writing on to see if she can figure out what he's been doing.

Nitara: How the heck did you get six from…

Ian: What?

Michael: Because he's smarter.

Ian: Because every time we extended the chain there's just two more minutes added.

Michael: He's smart.

Ian: Everybody, calls 2 people and that's just 2 minutes and everybody was at the same times. Unless. [pause] No, all right. [pause] Wait, I was thinking. You know what I was thinking? You know Mr T calls her first though? Wouldn't he start calling one minute before she was called?

Nitara: So each minute [inaudible] the conversation one minute then she calls the other person.

Ian: Yeah.

Nitara: Okay each minute so Mr T and that person were one minute. They hang up. She starts another one minute with someone else. They hang up. They start…

Ian: Watch watch watch.

Michael: Here's a good question. [pause] Who hung up first? [another pause while everyone looks at him]

Ian: [to Michael] No. [To group] What I was… Know what I was thinking? I was thinking Mr T would call one of them first so they would have one minute head start on the other person

Nitara: Ohhhh.

Ian: [inaudible]

Nitara: Kind of like ping pong, right?

Ian: Yeah
Nitara: Unless they did a conference call. Then the two they could be in the same conversation
Ian: He calls Ms J and Mrs W, we're not sure. It doesn't say Ms J *then* Mrs W.
Nitara: Yeah it says Mr T calls Ms J and Ms W.
There is some further debate about this.
Nitara: I think we should ask [puts up her hand to get Mrs. Shug's attention].
Jacob: Maybe they made a mistake.
Ian: Yeah, but then it's not our mistake, it's their mistake. If we're off by one minute we have our excuse.

[FIGURE 5]

Nitara is unable to get Mrs. Shug's attention. They decide to assume that it's not a conference call and start a new tree. It is challenging to keep track of all the numbers, so Nitara and Ian now both work on trees at the same time (Figure 5 is the tree Ian draws) so they can discuss and compare, Jacob continues to watch and contribute comments, while Michael is distracted. At some point Mrs. Shug comes by with a worksheet on integers for groups who finish the task early, and Jacob and Michael each start working on the worksheet while monitoring what the Nitara and Ian are doing. Nitara and Ian continue to work on the trees even once Mrs. Shug begins leading the class discussion.

Mrs. Shug has different groups share their answers. It seems all the groups who describe their solution have used the exponential pattern. Mrs. Shug then tells them they are wrong and she and the researcher explain to the class how each call is made one at a time, so that the first person Mr T called would be phoning someone at the same time that Mr. T makes his second call, and so on. Ian reacts immediately, slapping his hand on the desk, and telling first Nitara and then Mrs. Shug, "If we counted it, we were right!" Nitara and Ian immediately begin working on the trees again while Michael and Jacob watch. They continue working while Mrs Shug concludes the class discussion and gets the class to begin the transition to the next activity. She then joins the group and goes over their solution to the second question of the Snow Day task, concluding that they are on track to the correct answer. Nitara cheers and claps. Michael declares, "What a fantastic four!" and gives two thumbs up.

While Ian largely leads the group, Nitara also leads the discussion at times, and Jacob and Michael actively contribute ideas and comments throughout the session. They read the problem sheet aloud together very carefully when the task is introduced to the class and remain attentive to the details of the task throughout the session, re-reading the description and referring to it frequently. They also try to connect the task situation to the real world by referring to the list of teachers in Jacob's planner and what they know about the staffing of their school district. All of the group members are willing to ask questions, share their thinking, challenge other members' claims and calculations, and test their reasoning. They remain engaged in the task until the time for class discussion arrives, and then they draw on ideas raised during the class discussion which confirm that they are likely on the path to a workable solution at which point they begin working on the task again.

4. Discussion

Beginner's mind is characterized by an open awareness of the situation at hand, and a willingness to consider possibilities. Below I discuss some of the characteristics of beginner's mind consider the ways in which the group of Nitara, Ian, Jacob and Michael (NIJM) demonstrate beginner's mind while Eliana, Lucy and Geri (EGL) demonstrate expert's mind.

4.1 Level of awareness

In their work on the Snow Day task, NIJM's discussion raises a number of ideas. For the first question about how many calls are made in 10 minutes, they consider that the calls may be occurring in a ratio of 4:6, that there may be a connection with the actual number of teachers at their school, that is more

than one call happening in a minute as the tree goes on, and there may be a pattern of 2, 4, 8, 16, etc. For the second question, they discuss multiplying by 1000, working backwards from 1000, that each person can only make one call at a time and the implications of that, and that there could be conference calls.

Although Ian is often leading his group's discussion, there is no sense that he is the expert. All group members are contributing ideas and comments. When Ian himself offers an idea, it is as much to get their feedback as to convey what he is thinking. When he draws up trees, Nitara is also creating trees at the same time, and they are frequently checking each other's progress, while Jacob (and sometimes Michael) watches their work. No member of this group ever works separately from the others, and no one ever works ahead. They are aware of each other as contributors and work together.

In comparison, EGL's awareness is quite narrow. Early in the task, after rejecting the idea that there are 10 calls in 10 minutes because each call takes a minute, Eliana and Lucy are stuck. As Mason writes, being stuck is emotionally uncomfortable.

> A vague sense that no progress is being made begins to grow inside me. I become aware that I am not making progress, but the awareness is so diffuse that it takes the form of a gnawing than of anything which can be articulated. This awareness gradually increases to the point of articulation that "I am stuck". Still I remain at my desk. As the strength behind the statement "I am stuck" increases, the gulf becomes more tangible and less all-engrossing. At a certain threshold of intensity, it becomes possible to seek for assistance, in the form of a colleague, a text, or a change of activity. (1982, p. 23)

Eliana and Lucy choose to get out of being stuck by asking Geri what she got. While another student might have given them just enough of a hint to get them going again, Geri assumes that her solution is correct, and tells them what to do and the final answer. Eliana and Lucy have chosen what to look at (Berger, 1973) namely Geri's solution, and their attention remains there for the rest of the session. Any further writing, both on her sheet and on theirs, she does is in service of that explanation, to offer an alternate sketch, or to add more detail to her own work. She performs the necessary calculations, but she doesn't remain in the task. While none of the students are required to produce a "good copy" of their work for the tasks in this study, Geri still does the next best thing – she erases her initial work on the front of her task sheet, as if she is trying to offer a more pristine surface similar to a textbook example where the rough work is never visible (Pimm, 1987) only the steps that successfully lead to the correct answer. This may suggest her awareness of what mathematics "should" look like.

4.2 Possibilities

Related to awareness, beginner's mind contains the willingness to go off script, to ignore (at least temporarily) what *should* be done in favour of the possibilities of what *could* be done. This involves a different set of questions: "Does this make sense?" "What else might I try?" "What else do I notice?" What is particularly interesting about NIJM is that up until that moment that Ian says "Unless", they appear to have settled on a strategy that works, the exponential pattern. After all, they've just explained it to Mrs. Shug. However, they are not heavily invested in this strategy. Their awareness is focused on the task situation itself rather than just on their current solution. They are "ready for anything" (Suzuki, 1985).Thus, they are open to other ideas, a willingness to explore further, which means they are still thinking about the task even after it seems to be done, and there is no hesitation to address the "Unless' that Ian offers, a "[deviation] from expectation… which can lead to new sense-making" (Mason, 2014, p.4), and to start considering the new possibilities that may arise from this.

EGL's situation is more circumscribed. Eliana's awareness is expressed through her questions to Geri: "Did you get this?" "How did you do this?" "Where did you get this from?" There is no questioning of *why* Geri has chosen this particular strategy, only about the steps Geri has followed, and where particular numbers in her calculations come from. It is interesting that Geri does not ask Eliana or Lucy any questions or for any ideas. That she is able to get to work on and complete the Snow Day task so quickly suggests that perhaps she believes she has worked on a similar task before, and there is no need to

get any input from her group[2]. When Geri reaches an answer that satisfies her, she is done. She puts the task sheet away and starts working on something else. It is only Eliana and Lucy's questions that prompt her to pick it up again and add more detail in order to explain her ideas more fully.

4.3 Being one with the mathematics

Another sign of NIJM's beginner's mind is their high level of engagement in the task. They work throughout the period; even when Mrs. Shug hands out the integer worksheets, Nitara and Ian keep drawing their trees, with Jacob still observing them while he completes an integer worksheet. When Mrs. Shug begins the whole class discussion, Nitara and Ian only gradually pull away from their own work to listen. During the class discussion, when they realize they were on the right track with their solution path, the whole group begins working again, even though the class discussion has not ended yet. Later, when the rest of the class has handed in their sheets and started preparing for the next class activity, the members of NIJM are working with Mrs. Shug to discuss the second problem in the task further and their solution. They are so absorbed, they seem to be one with the mathematics.

Compare this behaviour to EGL's. Perhaps because Geri started work early and had a "correct" solution in front of her by the time Eliana and Lucy begin, the choice EGL makes is to be efficient, focused only on Geri's solution. Geri's role becomes teaching them the "right way" to do the task with Eliana seeking to understand and replicate her method, while Lucy, on the other hand, appearing to be satisfied by replication without understanding, because she doesn't "have time." Her mind is preoccupied by her upcoming band performance. EGL finishes early and move on to other school work. At the end of the class discussion, they hand in their papers without any further discussion of their solution and prepare for the next class activity. But there is a cost: "Because awareness opens doors and practice produces facility, we know that neither can be considered sufficient on its own. But the importance difference is that the presence of awareness makes practice meaningful, while practice alone can lead to boredom and nonretention. Awareness acts as glue to hold together the benefits of practice" (Gattegno, 1987, p. 108-9). Lacking sufficient awareness, it is not clear what the members of EGL, particularly Lucy, will have gained from the Snow Day task.

5. Conclusion

For many learners, mathematics is a subject that turns the popular saying "it's the journey, not the destination" upside-down. When the final product is privileged over the process of getting to that final product, there comes an expectation that mathematical understanding will be instantaneous and a valuing of compressed pathways. As a result, students who are motivated to perform mathematics correctly, like Eliana, may be haunted by the sense that they do not really understand what they are doing (Boaler, 2016). In the court of mathematics, they are always judged as wanting.

The beginner's mind is a ready mind, one that is alert and open. This kind of mind is an advantage for the mathematics learner. Rather than loading students with mathematical procedures as a way of coping with a packed school timetable, we as educators we would do well to encourage an open level of awareness, helping students develop and strengthen habits of mind that will enable them to explore, notice and think about mathematics and to deepen their level of understanding.

Disclosure statement

[2] When working on another task later in the larger study, Geri spends part of the session working separately from the rest of her group, having noted aloud that she has "done this before," trying to recall what she "did." It is interesting that on this occasion when Geri eventually presents her strategy to the rest of the group, Eliana and Rebekkah (another group member who was absent for the Snow Day task) are able to point out why it actually won't work, consoling her by saying that it is a good idea and they wished it would work because the diagrams are interesting.

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Figure Captions

Figure 1: Eliana and Lucy's initial work.

Figure 2: Geri's solution on the back of her task sheet.

Figure 3: Eliana and Lucy's final solution, with Geri's writing at the bottom.

Figure 4: Nitara and Jacob's initial solution tree.

Figure 5: Ian's final solution tree.